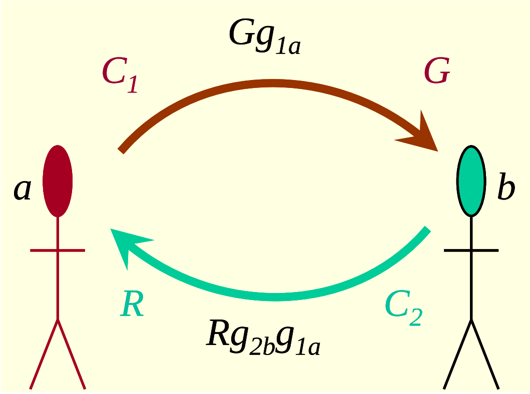
Cooperation without selection of partners

First player (a) offers a ‘gift’ to second player (b) that is proportional to gene g_1, which controls its propensity to make that gift: Gg_{1a}
(subscript a means that we consider g_1 expressed in a).
Second player (b) returns the favour by giving an amount that is proportional to what it received and to its gene g_{2b}: $Rg_{2b}g_{1a}$
Both actions involve proportional costs.



- First player’s payoff: $g_1 n\,(Rg_2 - C_1)$

Return from partner: Rg_1g_2
First step cost: $-C_1g_1$
Multiplicative noise: n

- Second player’s payoff: $g_1 n\,(G - C_2 g_2)$

Initial gift from partner: Gg_1
Second step cost: $-C_2g_2g_1$
Multiplicative noise: n

Now we consider genes’ strategies (as only genes survive in the long run)

- Average first player’s payoff: $Ng_1\,(Rg_{2m} - C_1)$ (1a)
- Average second player’s payoff: $NPg_{1m}(G - C_2g_2)\,s$ (1b)

Probability of being selected as second player: s
Size of the group: P
Number of interactions initiated by one individual each year: N

Both g_1 and g_2 are expected to drop down to zero.

Cooperation with selection of partners

- Average first player's payoff:

$$Ng_1(pRg_{2M} + (1-p)Rg_{2m} - C_1) \qquad (2a)$$

Probability of interacting with remembered friend: p (measures exploitation *vs.* exploration)
Average cooperativeness of remembered friends: g_{2M}

- Average second player's payoff:

$$NPg_{1m}(G - C_2 g_2)\, s(g_2/g_{2m}) \qquad (2b)$$

Derivative of (2b) :

$$NPg_{1m}\,[-C_2 s(g_2/g_{2m}) + (G - C_2 g_2)\, s'(g_2/g_{2m})\,/g_{2m}] + NP\,\partial g_{1m}/\partial g_2 \times (G - C_2 g_2) s(g_2/g_{2m})$$

At equilibrium: $g_2 = g_2^* = g_{2m}$

Cooperative strategy:

$$g_2^* = \frac{G}{C_2\left(1 + \frac{s(1)}{s'(1)}\right)} \qquad (3)$$

First step's average payoff becomes: $Ng_1^*(Rg_2^* - C_1)$

Condition for cooperation:

$$\frac{GR}{C_1 C_2} > 1 + \frac{s(1)}{s'(1)} \qquad (4)$$

Payoff of cooperative individuals:

$$NPg_{1m} G \frac{s(1)}{1 + \frac{s'(1)}{s(1)}}$$

Payoff of uncooperative individuals: $NPg_{1m} G s(0)$

Global payoff of the population

$$PNg_{1m}[(Rg_{2m} - C_1) + (G - C_2 g_{2m})\, s(1)P]$$

$$= PNg_{1m}[(R - C_2)\, g_{2m} + (G - C_1)]$$

$$PNg_{1max}[(R - C_2)\, g_2^* + (G - C_1)]$$

$$= PNg_{1max}[(R - C_2)\, G / (C_2(1 + s(1)/s'(1))) + (G - C_1)]$$

$$= PNg_{1max}\left[\left(\frac{GR}{C_1 C_2\left(1 + \frac{s(1)}{s'(1)}\right)} - 1\right)C_1 + \frac{G}{\left(1 + \frac{s'(1)}{s(1)}\right)}\right]$$

There is a nett global profit, resulting from wealth production.