Why altruism?

Social emergence in complex systems

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Altruism and social life

La Liberté guidant le peuple, Delacroix, 1830

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"Daily plebiscite" [Renan, 1882]

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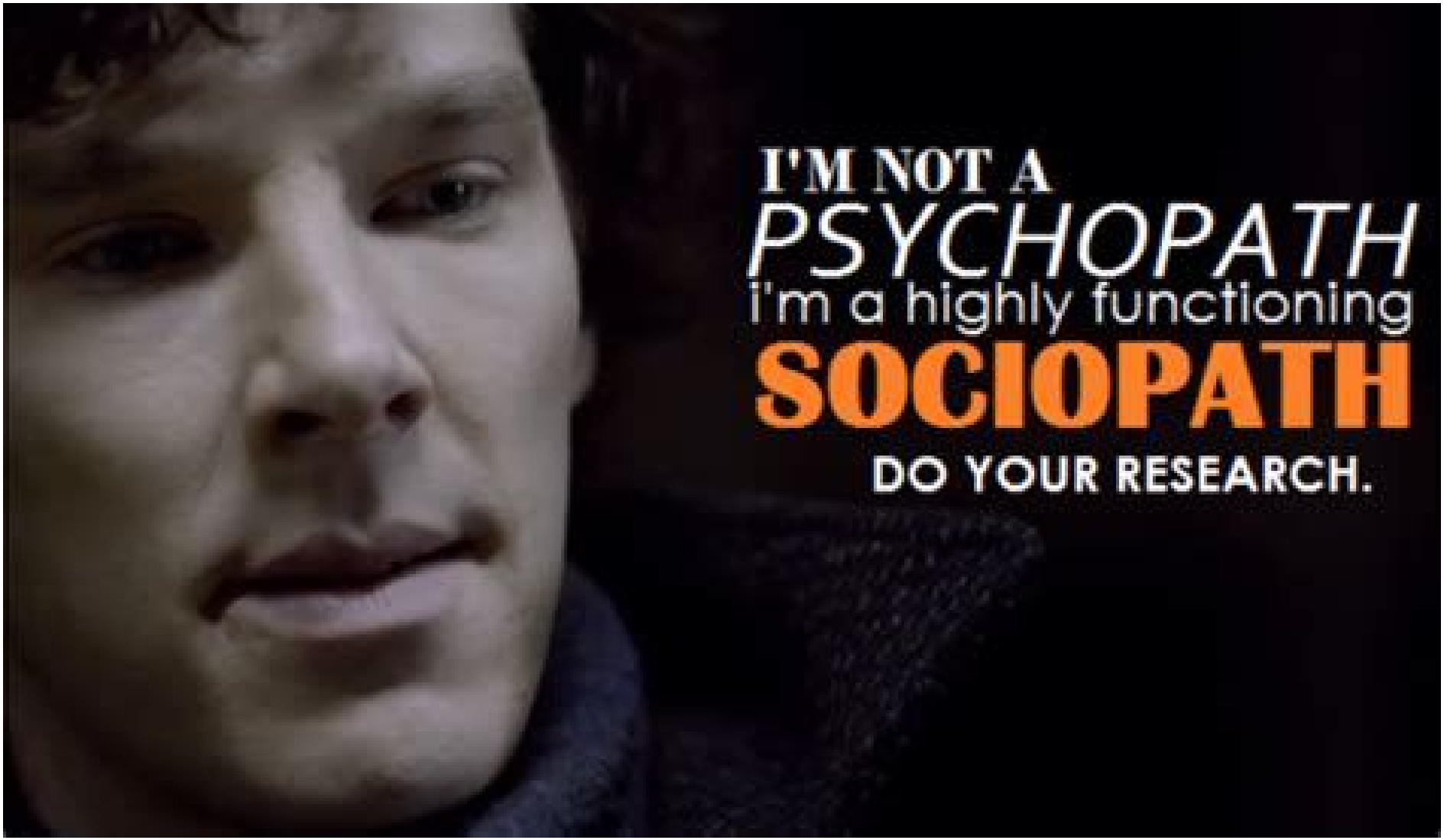
Thanks the tireless (often unpaid) work of some, the internet is an incredible source of knowledge

Why altruism?



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Pack hunting

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Assumptions (public good):

Collectively shared benefit:

$$B(x_1, x_2...) = \frac{\sum_{j=1}^{n} b*x_i}{n}$$

Quadratic individual cost:

$$C = \frac{1}{2}c * x^2$$

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Total payoff if I invest x and others x_0:

$$F(x, x_0^{-1}) = \frac{b*x+(n-1)b*x_0}{n} - \frac{1}{2}c * x^2$$

Optimal investment:

$$x^* = \frac{1}{n} * \frac{b}{c} > 0$$

Collective hunting with effort x^* is optimal for each dog

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Definition (biology)

In the previous example, each dog obtains:

$$\frac{bn \times (\frac{1}{n} * \frac{b}{c})}{n} - \frac{1}{2}c \times (\frac{1}{n} * \frac{b}{c})^2 = \frac{1}{n}\frac{b^2}{c}(1 - \frac{1}{2n}) > 0$$

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An individual engages in **altruisic behavior** if:

- the behavior is costly to the individual: $-C$
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Natural selection: why should altruistic behavior evolve?

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Examples of non-human altruism (?)

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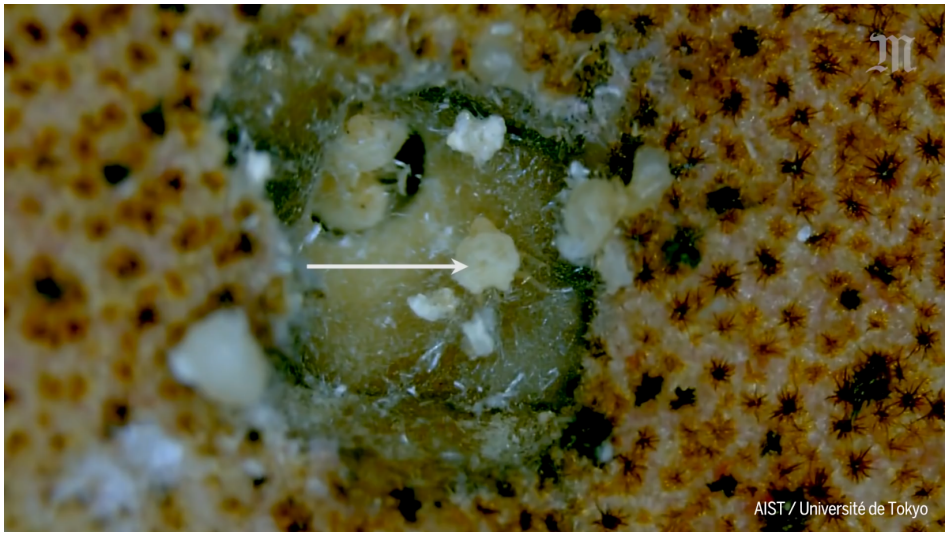

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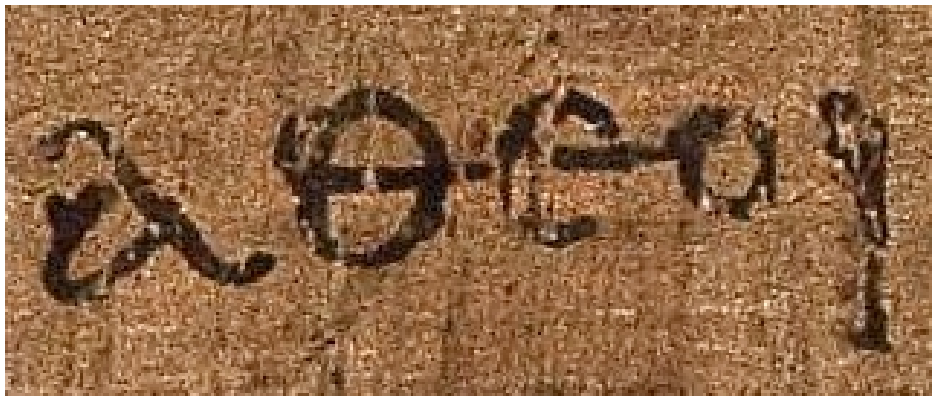
The tragedy of the commons



Koine Greek

Wikipedia

Yosemite, USA

Open source initiative

The tragedy of the commons and (biological) altruism

$$x^* = \frac{1}{n}\frac{b}{c} \xrightarrow[n\to\infty]{} 0$$

Tragedy of the commons [Hardin, 1968]

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A largely-shared public good can only be stable if (many) individuals invest $x > x^*$, hence more than they get back:

$$c \times x = C > B = \frac{b \times \sum_{i=1}^{n} x_i}{n} \tag{1}$$

Individuals who invest $x > x^*$ display **altruistic** behavior

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Nash equilibrium

Game theory: bag of analytical tools to help understand the phenomena that we observe when decision-makers interact
[Osborne and Rubinstein, 1994]

Assumptions:

- *rational* decision-makers
- who reason *strategically*

$$x^*$$

$$F(x, x_0^{-1}) = \frac{b*x+(n-1)b*x_0}{n} - \frac{1}{2}c*x^2$$

Nash equilibrium

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Nash equilibrium: steady state of the strategic game (best reply to itself)

For two players 1 and 2 with identical strategy set $\mathcal{S}$, a strategy $S \in \mathcal{S}$ is Nash if both perform worse when shifting to another strategy T:

$$\forall i \in \{1,2\}, \forall T \in \mathcal{S},\ \pi_i(S,S) \geq \pi_i(T,S)$$

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Bonus slides: normal form game

Example: Prisoner's dilemma (if we have the time — otherwise tomorrow).

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Back to the tragedy of the commons

- Hunting: investing effort x^* is Nash
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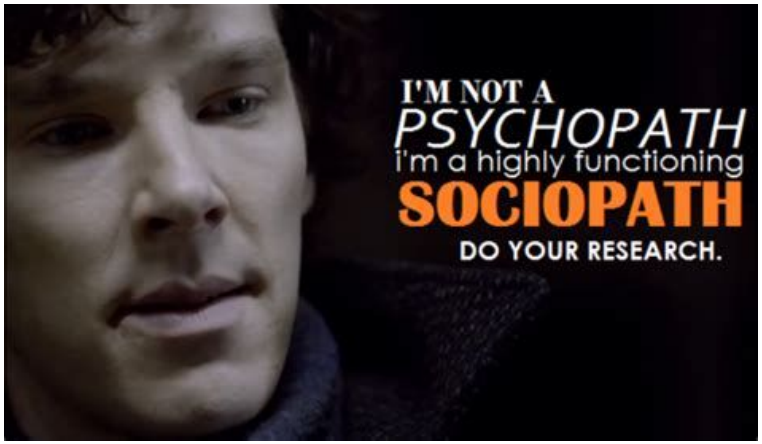
$\rightarrow$ *Collective benefit is not an explanation* ! (tautological)



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Wait a minute:

- We (sometimes) behave altruistically
- Nash equilibria emerge from individual cost-benefit analyses

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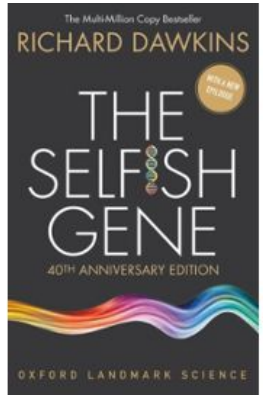
$\rightarrow$ OK perhaps, but *evolution favours "selfish" genes*



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Evolutionary game theory

→ *Emergence of collective behavior due to evolution*

Basic principles ("replicator dynamics"):

- individuals in a biological population are characterized by different strategies
- they play the game
- they reproduce according to payoffs obtained in the game ("fitness")
- strategy is inherited
- alternative strategies may occasionally arise due to mutation

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Evolutionary Stable Strategy (ESS): Nash equilibrium that can't be invaded by mutants. $S \in \mathcal{S}$ is an ESS iff:

- S is a Nash equilibrium
- Every other best reply to $T \neq S$ satisfies: $\pi(T, S) > \pi(T, T)$

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Tree height

Why are trees so tall?

- Trees feed off sunlight (photosynthesis) — $-B$
- Maintaining a long trunk is costly (risk of breaking, pumping water...) — $-C$

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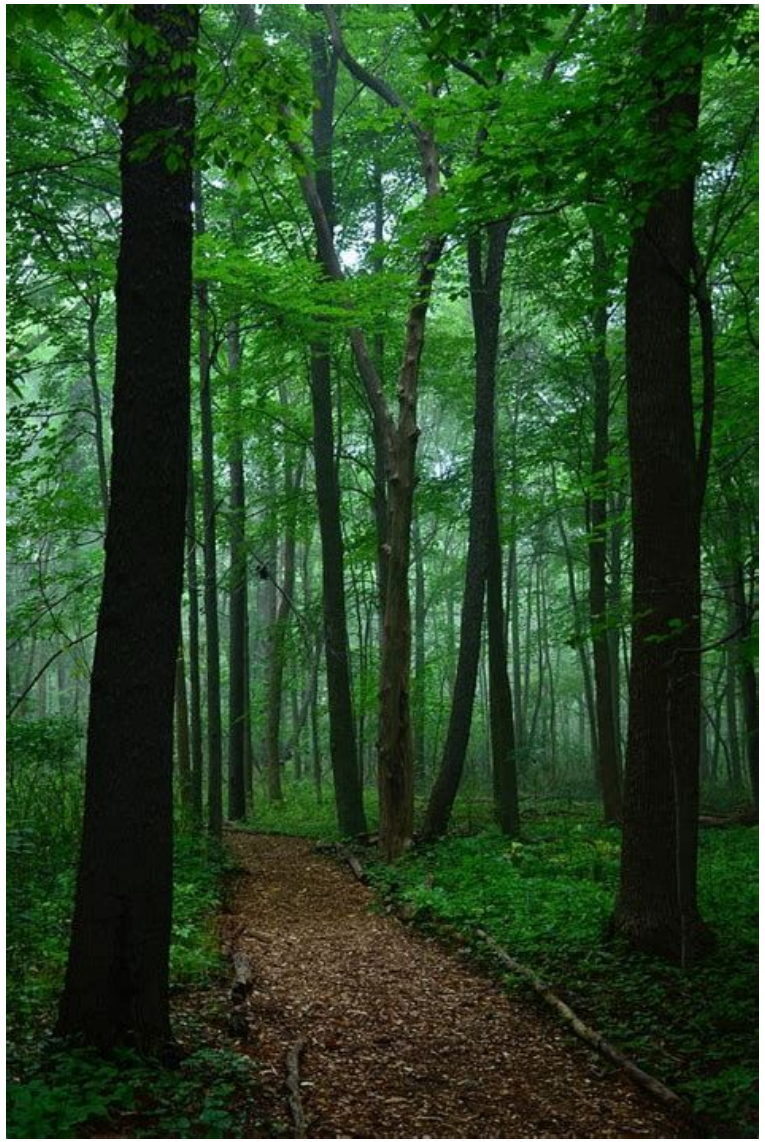
- Why may tall trees be an ESS?



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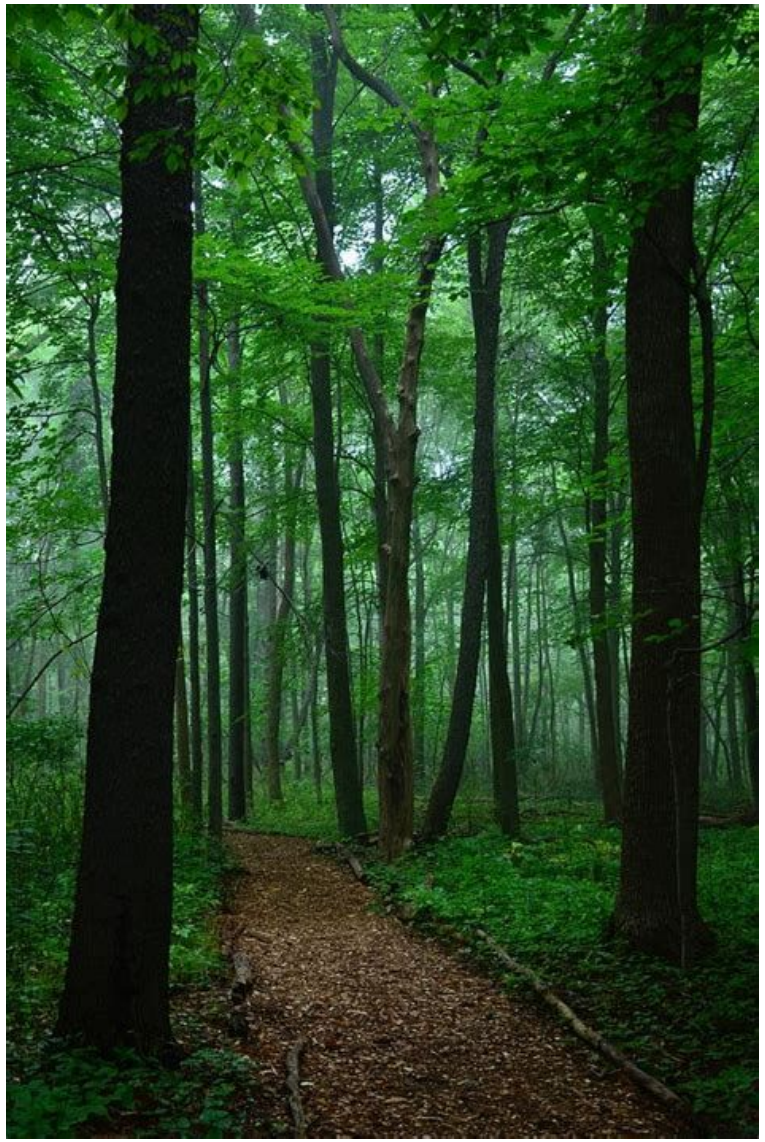

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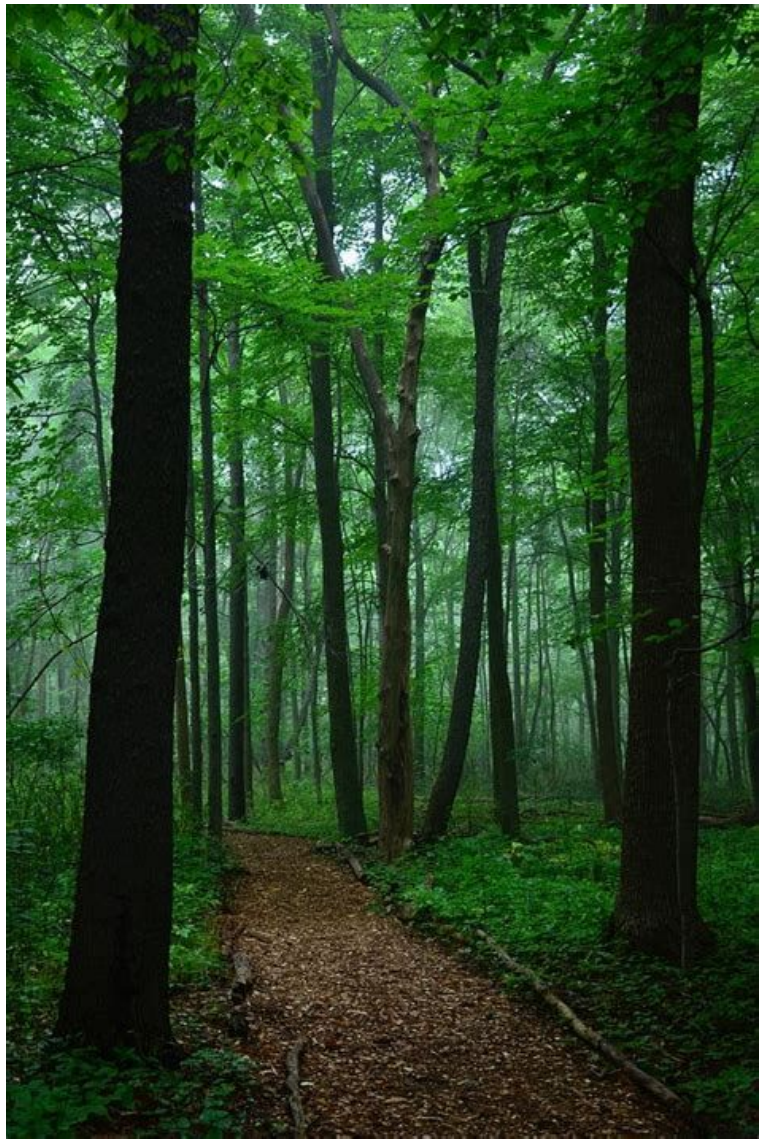
To remember: reasoning in terms of (potential) ESS and **mutants**

NB: Trunks amount for a huge amount of the biomass



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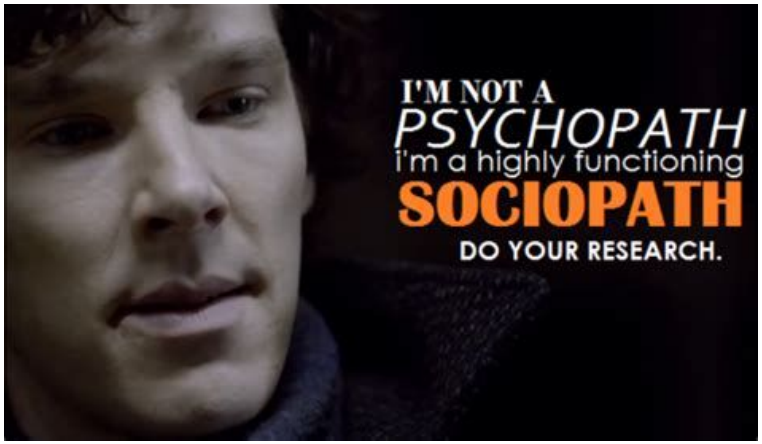
Evolutionary game theory and altruism

Tomorrow: when is altruism an ESS? E.g. when will contributing more than x^* be *selected*?



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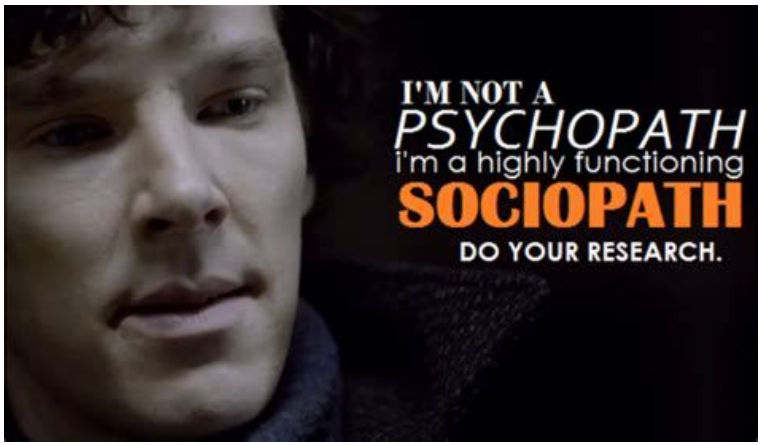

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Bonus slides: back to the normal form game (Prisoner's Dilemma)

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